

**Seminar of the Chair of Optimization and Control**  
**under prof. Stanisław Migórski**  
**winter semester 2018-2019, Thursday, 10:15-11:45, room 1177**

**October 4, 11 2018**

Yunru Bai, Sensitivity of Hemivariational Inequalities and Applications

**October 18, 25 2018**

Shengda Zeng, A mixed hemivariationalvariational problem and applications

The talk was based on the paper:

Andaluzia Matei, A mixed hemivariationalvariational problem and applications, Computers & Mathematics with Applications, <https://doi.org/10.1016/j.camwa.2018.08.068>

Abstract: We consider an abstract system with Lagrange multipliers which consists of a hemivariational inequality and a variational inequality. The hemivariational inequality is governed by a hemicontinuous, generalized monotone, possibly nonlinear operator as well as by a bilinear form. This bilinear form also governs the variational inequality. We are looking for a pair solution in a subset of a product of two real reflexive Banach spaces. In order to illustrate the applicability of the abstract results, two examples in terms of PDEs are delivered. Each example is related to the weak solvability of a boundary value problem arising in contact mechanics.

**November 8, 2018**

Leszek Gasiński, Note on local minimizers  $W^{1,p}$  versus  $C^1$

**November 15, 22 2018**

Weimin Han, Numerical Analysis of Variational and Hemivariational Inequalities with Applications

Course plan:

- A brief introduction of variational and hemivariational inequalities arising in contact mechanics;
- A review of well-posedness results of sample variational and hemivariational inequalities;
- Numerical analysis of variational and hemivariational inequalities: description of numerical methods, convergence analysis, error estimation;
- Discussion of future research topics.

**November 29, December 6, 13 2018**

Zhenhai Liu, Partial Differential Operators and Regularization of Quasi-hemivariational Inequalities

Course plan:

- Properties of elliptic partial differential operators (introduction of several kinds of abstract monotone type operators, proofs in details for the (S+)-property and pseudomonotonicity of elliptic partial differential operators)
- Properties of nonlocal p-Laplace operators (introduction of nonlocal operators, fractional spaces, proofs in details for the strong monotonicity for nonlocal operators)
- Existence of solutions for elliptic variational hemivariational inequalities. The operators involved are taken to be multivalued and noncoercive.
- Regularization of quasi-hemivariational inequalities (An ill-posed quasi-hemivariational inequality with contaminated data can be stabilized by employing the elliptic regularization. Establish the regularization theory for quasi-hemivariational inequalities with set-valued monotone maps or generalized pseudo-monotone maps. Applications to elliptic quasi-hemivariational inequalities and nonlocal quasi-hemivariational inequalities)
- Discussion on selected recent papers and current trends in the area.

**January 3, 2019**

Michał Jureczka, A nonsmooth static frictionless contact problem with locking materials

The talk was based on the paper:

Mircea Sofonea, A nonsmooth static frictionless contact problem with locking materials, Analysis and Applications, doi: 10.1142/S0219530518500215

Abstract: We study a new mathematical model which describes the equilibrium of a locking material in contact with a foundation. The contact is frictionless and is modeled with a nonsmooth multivalued interface law which involves unilateral constraints and subdifferential conditions. We describe the model and derive its weak formulation, which is in the form of an elliptic variational-hemivariational inequality for the displacement field. Then, we establish the existence of a unique weak solution to the problem. Next, we introduce a penalty method, for which we state and prove a convergence result. Finally, we consider a particular version of the model for which we prove the continuous dependence of the solution on the bounds which govern the locking and the normal displacement constraints, respectively. We apply this convergence result in the study of an optimization problem associated to the contact model.

**January 10, 2019**

Anna Kulig, A class of generalized evolution variational inequalities in Banach spaces

The talk was based on the paper:

Yi-bin Xiao, Nan-jing Huang, Yeol Je Cho, A class of generalized evolution variational inequalities in Banach spaces, *Applied Mathematics Letters* 25 (2012) 914920, doi:10.1016/j.aml.2011.10.035

Abstract: In the present paper, a class of generalized evolution variational inequalities arising in a number of quasistatic frictional contact problems for viscoelastic materials is introduced and studied. Using Banach's fixed point theorem, the existence and uniqueness theorem of the solution for the generalized evolution variational inequalities is proved under some suitable assumptions.

**January 17, 2019**

Michał Jureczka, A nonsmooth static frictionless contact problem with locking materials, part. 2.

**January 24, 2019**

Krzysztof Bartosz, Basic principles of virtual element methods

The talk was based on the paper:

L. Beirão da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L. D. Marini, and A. Russo, Basic principles of virtual element methods, *Math. Models Methods Appl. Sci.* 23 (2013), 199-214

Abstract: We present, on the simplest possible case, what we consider as the very basic features of the (brand new) virtual element method. As the readers will easily recognize, the virtual element method could easily be regarded as the ultimate evolution of the mimetic finite differences approach. However, in their last step they became so close to the traditional finite elements that we decided to use a different perspective and a different name. Now the virtual element spaces are just like the usual finite element spaces with the addition of suitable non-polynomial functions. This is far from being a new idea. See for instance the very early approach of E. Wachspress [A Rational Finite Element Basic (Academic Press, 1975)] or the more recent overview of T.-P. Fries and T. Belytschko [The extended/generalized finite element method: An overview of the method and its applications, *Int. J. Numer. Methods Engrg.* 84 (2010) 253304]. The novelty here is to take the spaces and the degrees of freedom in such a way that the elementary stiffness matrix can be computed without actually computing these non-polynomial functions, but just using the degrees of freedom. In doing that we can easily deal with complicated element geometries and/or higher-order continuity conditions (like C1, C2, etc.). The idea is quite general, and could be applied to a number of different situations and problems. Here however we want to be as clear as possible, and to present the simplest possible case that still gives the flavor of the whole idea.